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Numerical evaluation of current paths in disordered media

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Abstract. We present numerical results for the current flowing though a disordered block of material. The results show the current flowing in well-defined channels with a disorder and energy dependent width. The correlation of the current shows a negative dip at short separations.

1. Introduction

The problem of localization in random media has been a topic of interest since Anderson's work [1]. Despite more than three decades work on the topic, and many results (for a review see Kramer and MacKinnon [2]), there is still no intuitive description of the localization transition.

Recently work has been done to try to describe transmission through disordered media within a physical picture. Chalker [3] has described localization in terms of the evolution of the population of different channels as a consequence of backscattering and diffusion. Bell and MacKinnon [4] have tried to describe the transition in a real-space renormalization picture. Pendry [5] has described the importance to the conductance of strings, 'necklaces', of resonant states.

Despite these models we have no good picture of the form of the current in disordered systems. Something is known of the form of the wavefunctions [6–10], especially because of the interest in the quantum Hall effect in magnetic fields. The wavefunctions have been found to be fractal [11, 12] and in fact multifractal near the localization transition [13–19]. The fractal dimension (within two-spatial dimensions) is much less than two so that the states must have a spaghetti like structure [2]. Dasgupta *et al* have studied the wavefunction in a two-dimensional random system with leads, and have found evidence of necklace states [20].

Since most experiments, and many of the analytical approaches are concerned with the transmission properties of systems, it seems of interest to see how the current flows through a disordered system. The problem is of interest even in the absence of a localization transition, so we consider the simplest system, a two-dimensional lattice with a random onsite potential.

We present here the result of numerical calculations of current paths through disordered blocks. The results show the current typically flowing in channels with a characteristic disorder and energy dependent width. We have examined the spatial correlation of the current flow, and find that there is a region of negative correlation where, near to a region of strong current flow, the current is typically less than the average further away. Since the current is conserved the figures emphasize the connections across the system, rather than emphasizing the resonances where the wavefunction is largest.

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2. Method

We have used the recursive Green function method [21, 22] to calculate the conductance of two-dimensional blocks of disordered material connected to ordered leads with the same Fermi energy, but zero disorder. We have calculated the current following Baranger *et al* [23] and the ambiguity in the calculation of the current discussed there applies to our calculation.

We have used a Hamiltonian defined on a square lattice with nearest neighbour coupling $V = -1$ and random onsite elements taken from a Gaussian distribution with zero mean and standard deviation σ. Energies are specified relative to the band edge, and we discuss energies below the centre of the band, at four units above the band-edge. The behaviour of the model is statistically symmetric for energies above and below the centre of the band.

Since the calculation is performed on a square lattice, with the Green function defined between nodes of the lattice. The current is therefore defined on the lines of the lattice. For the sake of a convenient graphical representation we define the current at each node as the sum of the currents in the four lines meeting at that point.

3. Results

Figures 1, 2 and 3 show the current pattern for three samples. For each instance of the disorder a scan was made of different energies to find the maxima in the conductance. The current was calculated for the higher of these maxima, and some representative results are shown in the figures. In so far as a current can flow through the system, the patterns with different energies and different realizations of the disorder look qualitatively similar [24]. We have examined plots of the corresponding random potentials, and have found no visible correlation between the potential and the current path. The potential has no correlation from site to site, so that the relation between the current paths and the potential is more complex than simply following valleys of the potential.

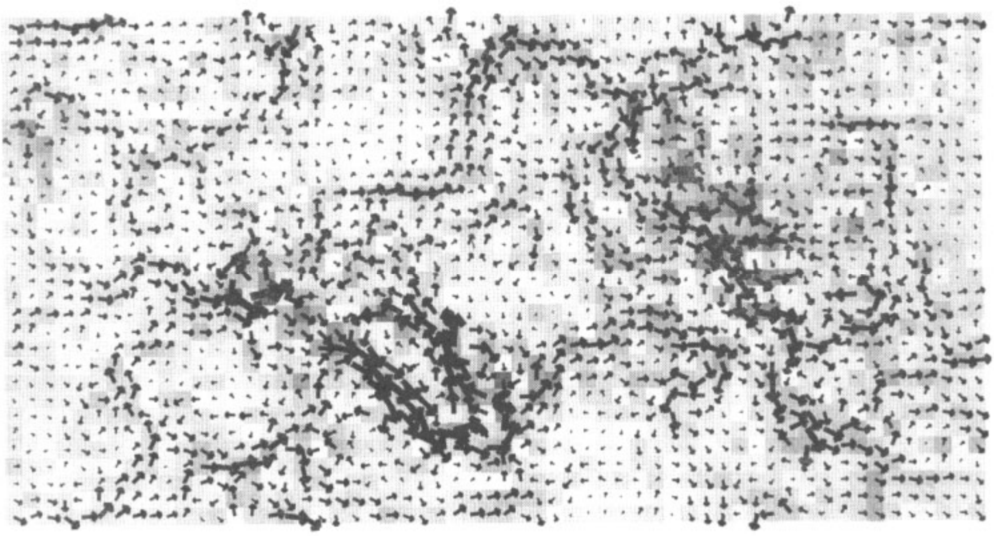

Figure 1. Current in a 30×60 block of disordered material with disorder $\sigma = 1$ and energy $E = 1.3$ The conductance was $1.4e^2/h$.

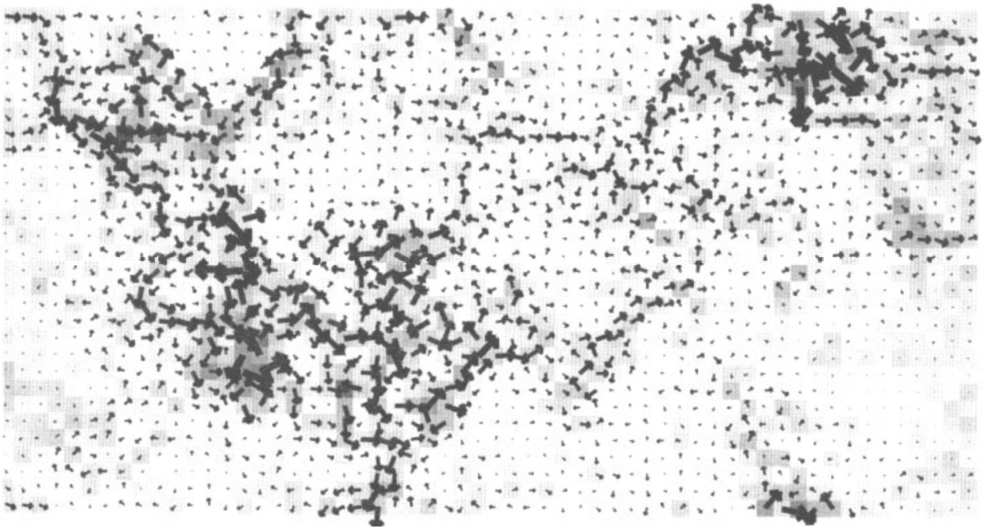

Figure 2. *Current in a 30 × 60 block of disordered material with disorder $\sigma = 2$ and energy $E = 2.7$. The conductance was $0.66e^2/h$.*

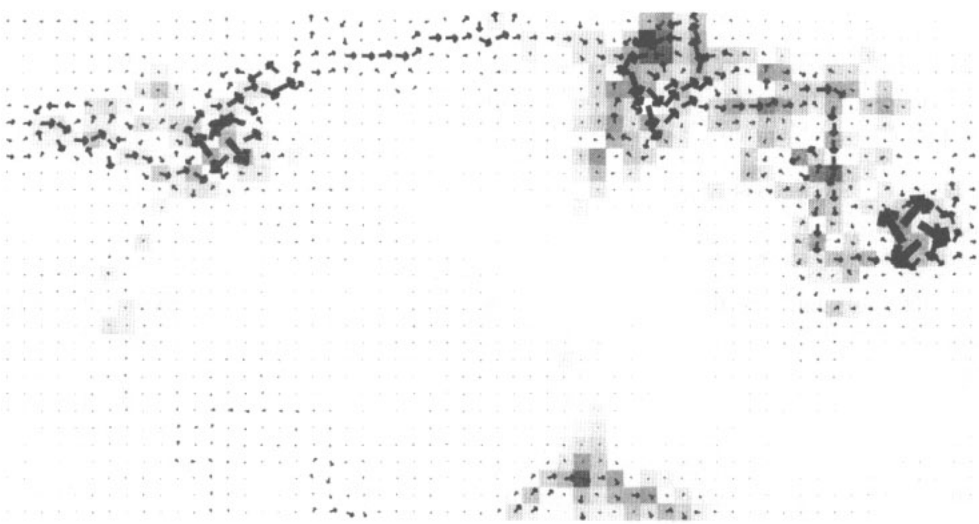

Figure 3. Current in a 30 × 60 block of disordered material with disorder $\sigma = 4$ and energy $E = 2.8$. The conductance was $1.4e^2/h$.

All the plots show the current distribution by arrows whose area is proportional to the local current density. For clarity the length and the width of the lines making up the arrows are each proportional to the square root of the current. While a streamline plot is possible, such plots obscure the periodic boundary conditions, and do not have the same dynamic range as the arrow plot. The shading indicates the local density of states. Current is injected from the left hand lead in each case. Periodic boundary conditions have been used in the other direction.

In an infinite localized system, there would in general be no state at a given position and energy. Since the systems here are connected to leads, the states are broadened, and the density of states recorded here reflects both the local states, and their connection to the leads. Accordingly, as figure 3 shows clearly, there is an enhanced density of states near the leads, and near the conducting path. Of course at other energies, there will be highly localized states in the centre of the sample, so that the mean density of states will be similar both in the middle and at the ends of the sample. Figure 3 shows the current flowing through 3 resonances aligned roughly in a straight line, in accord with Pendry's picture of necklace states [5]. Since the resonances are asymmetric there is a circulating current in each one.

Just as for an infinite system there is in general be no state at a given position and energy, there is in general no conductance across an infinite system. We expect, but cannot however tell from results on a finite system, that for those exceptional energies where a current can flow, the current will look generically like the current in our results.

The results show a clear tendency for the current to flow in channels with a well defined width. To help quantify this we have looked at the spatial correlation of the current. To do this we have taken random instances of the disorder at fixed energy, and we have calculated the correlations in the current. Labelling the nodes of the square lattice by their coordinates I and J and the current at the node by J_{IJ}, as

$$C_N = \frac{\langle J_{IJ} J_{IJ+N} - \langle J_{IJ}\rangle^2\rangle}{\langle J_{IJ} J_{IJ} - \langle J_{IJ}\rangle^2\rangle}.$$

The averages are taken over a single row of sites, and C_N then averaged over different rows and samples. The results in this paper are all taken with the current flowing through the sample in the X direction, and with correlations taken in the Y direction. Similar results are obtained taking the correlation of the Y component of the current in the X direction as for the X component of the current in the Y direction. Similar results are also obtained taking the correlations of the currents in the individual wires.

Figure 4 shows the resulting correlations, which display several interesting features:

(i) the correlation of the Y current, that is the correlation of the component of the current along the line it is flowing, is relatively long range compared with the correlation of the X component of the current;

(ii) the correlation of the Y current is relatively weakly affected by the disorder and energy;

(iii) the correlation of the X current shows a minimum at a short separation before rising towards its asymptotic value as the separation is increased;

(iv) the mimimum moves to a shorter separation as either the disorder or the energy is decreased.

We have calculated the localization length at the same energies and disorders as the correlations; the results are shown in table 1. We see that in each case we are looking at behaviour on a length scale small compared with the localization length. Except for the weakest disorder we are using system sizes which are large—in the case of the strongest disorder very large—compared with the localization length.

The correlations of the Y current indicate a tendency for the current to flow in straight lines. As figure 3 especially shows clearly, there is a tendency for the current to flow parallel to the either the X or the Y direction.

The negative dip in the correlation of the X current can be understood in a number of complementary ways. First, in the spirit of Chalker's work [3], consider the current

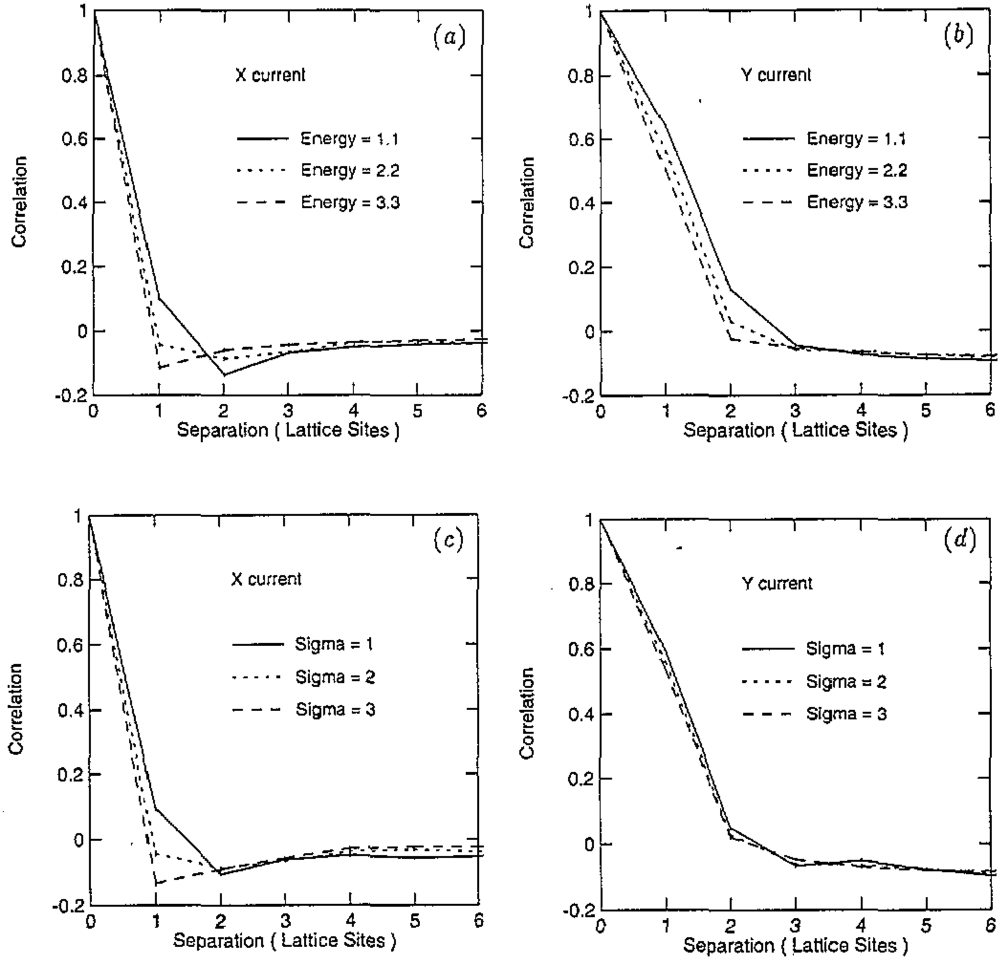

Figure 4. Correlation of current as a function of separation in the Y direction, that is perpendicular to the current flowing through the wire, for different disorders and energies. Error bars are plotted, but are negligible.

Table 1. Localization lengths.

Disorder	Energy	Localization length
1	2.2	77.5
2	1.1	10.5
2	2.2	15.7
2	4.4	17.7
4	2.2	3.3

flowing along one of the links of the lattice. Current will flow on into the sample, and some of the current will be backscattered, producing a contribution to the current in the opposite direction to the current we just considered. The backscattered current will be on average greatest in the links close to the one considered, leading to the negative dip in the correlation. Second, given that there are circulating currents, especially around resonances, these will produce adjacent currents flowing in opposite directions, leading to a negative dip in the correlation. Thirdly, if channels repel one another—consistent with the existence of well defined channels—there will be a negative correlation at short ranges.

The spatial spread of the backscattered current will depend on the mean free path of the particles. At higher disorders the mean free path will be shorter, so the backscattered current is in general closer to the injected current, and the minimum correlation is at a shorter distance. The mean free path is also expected to fall as the energy is increased—consistent with the behaviour of the correlations—since an argument due to Ioffe and Regel [25] suggests that the mean free path cannot be shorter than the wavelength of the particles, which we expect to fall as the energy increases.

4. Summary

We have presented results for the current flowing in disordered systems. The results show the current flowing in channels with a well defined width. For large disorder, we find the conducting states to have a necklace form as predicted by Pendry [5]. The spatial correlation of the current shows the effect of backscattering producing a negative short-range correlation. The behaviour of the short-range correlation is consistent with what we expect qualitatively for the mean-free path.

References

[1] Anderson P W 1985 *Phys. Rev.* **109** 1492
[2] Kramer B and MacKinnon A 1994 *Rep. Prog. Phys.* to be published
[3] Chalker J T and Bernhardt M 1993 *Phys. Rev. Lett.* **70** 982
[4] Bell P M 1993 *PhD Thesis* (University of London)
[5] Pendry J B 1987 *J. Phys. C: Solid State Phys.* **20** 733
[6] Aoki H 1983 *J. Phys. C: Solid State Phys.* **16** L205
[7] Aoki H 1984 *J. Phys. C: Solid State Phys.* **17** 1875
[8] Ohtsuki T, Kramer B and Onon Y 1992 *Solid State Commun.* **81** 477
[9] Ono Y, Ohtsuki T and Kramer B 1989 *J. Phys. Soc. Japan* **58** 1705
[10] Ono Y, Ohtsuki T and Kramer B 1991 *J. Phys. Soc. Japan* **60** 270
[11] Aoki H 1986 *Phys. Rev.* B **33** 7310
[12] Ohtrsuki T and Ono Y 1989 *J. Phys. Soc. Japan* **58** 956
[13] Pook W and Janssen M 1991 *Z. Phys.* B **82** 295
[14] Huckestein B and Schweitzer L 19092 *Physica* A **191** 401
[15] Fastenrath U, Janssen M and Pook W 1992 *Physica* A **191** 401
[16] Chalker J T, Daniell G J, Evangelou S N and Nahn I H 1993 *J. Phys.: Condens. Matter* **5** 485
[17] Basu C and Mookerjee A 1992 *J. Phys.: Condens. Matter* **4** 2857
[18] Thakur P K, Basu C, Mookerjee A and Sen A K 1992 *J. Phys.: Condens. Matter* **4** 6095
[19] Basu C, Mookerjee A, Sen A K and Thakur P K 1991 *J. Phys.: Condens. Matter* **3** 9005
[20] Dasgupta I, Saha T and Mookerjee A 1993 *Phys. Rev.* B **47** 3097
[21] Sols F, Macucci M, Ravaioli U and Hess K 1989 *J. Appl. Phys.* **66** 3892
[22] MacKinnon A 1985 *Z. Phys.* B **59** 385
[23] Baranger H U, DiVicenzo D P, Jalabert R A and Stone A D 1991 *Phys. Rev.* B **44** 10637
[24] We have examined around one hundred current patterns for each of the characteristic values of the parameters.
[25] Ioffe A F and Regel A R 1960 *Prog. Semicond.* **4** 237